

HW8 , Math 531, Spring 2014

Ayman Badawi

QUESTION 1. Prove that Z_{48} is ring isomorphic to $Z_3 \times Z_{16}$.

QUESTION 2. assume that I, J, K are ideals of R and $I = J \cup K$. Prove that $I = K$ or $I = J$

QUESTION 3. Let $R = Z[\sqrt{10}] = \{a + b\sqrt{10}\}$. Then R is an integral domain (You do not need to prove this, but if you need to know why? just observe that R is a subring of $\mathbb{R}$ (the set of real numbers)).

(i) Prove that 2, 3, $4+\sqrt{10}$, $4-\sqrt{10}$ are irreducible elements of R.

(ii) Prove that 2, 3, $4+\sqrt{10}$, $4-\sqrt{10}$ are not prime elements of R.

(iii) Prove that R is not a unique factorization domain. [Hint: observe that $6 = 2 \cdot 3$ and $6 = (4-\sqrt{10})(4+\sqrt{10})$], or just observe that since some irreducible elements of R are not prime elements, then R cannot be a UFD]

QUESTION 4. Let $n < \infty$ and R be a commutative ring with 1. Suppose that $P_1, ..., P_n$ are distinct prime ideals of R and I is a proper ideal of R such that $I \subseteq \cup_{i=1}^{n} P_i$. Prove that $I \subseteq P_k$ for some $1 \le k \le n$.[Hint: Let m be the least integer, $1 \le m \le n$ such that $I \subseteq \cup_{i=1}^{m} P_i$. If $m = 1$, then you are done. Hence assume that $2 \le m \le n$. Then for each $1 \le k \le m$, there is an $a_k \in I \setminus \cup_{i=1, i\neq k}^{m} P_i$. Now let $x = a_1 + a_2 a_3 \cdots a_m$. Clearly $x \in I$. Show $x \notin \cup_{i=1}^{m} P_i$, a contradiction.]

QUESTION 5. Let R and S are commutative rings with one.

(i) Let $I \subseteq J$ be proper ideals of R. Prove that $\frac{J}{I}$ is a prime ideal of R/I if and only if J is a prime ideal of R .

(ii) Let f be a ring epimorphism from R onto S, and $ker(f) \subseteq J$ be proper ideals of R. Prove that $f(J)$ is a prime ideal of S if and only if J is a prime ideal of R.

(iii) Let f be a ring epimorphism from R onto S. Let D be a prime ideal of S. Prove that $D = f(L)$ for some prime ideal L of R such that $Ker(f) \subseteq L$.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`